

GCSE Maths – Ratio, Proportion and Rates of Change

Compound Growth and Decay

Notes

WORKSHEET



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Compound Growth and Decay

Growth

Growth is when an original value is **increased** by a percentage.

An example is the pet health insurance sector for the United States. Gross premiums were at \$1.56 billion in 2019, and increased by 27.5% to reach \$1.99 billion in 2020.

Compound interest is a common example of **growth**. We will learn how to calculate **compound interest** later.

Growth can be calculated using a formula:

$$N = N_0 \times \left(1 + \frac{\text{percentage}}{100}\right)^n$$

N = amount after time period

N₀ = original amount

n = number of periods (this might be days / minutes / years)

Example: The population of salmon in a fishery is 5000, it increases by 10% each year. How many salmon will there be after 1 year? How many salmon after 5 years?

1. Use the formula with $n = 1$.

N = population after 1 year, $N_0 = 5000$, percentage = 10%

$$\text{Population} = 5000 \times \left(1 + \frac{10}{100}\right)$$

$$\text{Population} = \mathbf{5500 \text{ salmon}}$$

2. Use the formula with $n = 5$.

N = population after 5 years, $N_0 = 5000$, percentage = 10%

$$\text{Population} = 5000 \times \left(1 + \frac{10}{100}\right)^5$$

$$\begin{aligned} \text{Population after 5 years} &= 8052.55 \text{ salmon} \\ &= \mathbf{8053 \text{ salmon}} \text{ (to the nearest whole number)} \end{aligned}$$

The salmon population grew by 500 in the first year.

However, the population grew by different amounts each year due to the changing initial population.

For example, after the 2nd year, population = $5500 \times \left(1 + \frac{10}{100}\right) = 6050$.

*This is an increase of $6050 - 5500 = 550$ from year 1. This shows we **cannot** continually add $n \times 500$ to find the population after n years.*



Decay

Decay is when an original value is **decreased** by a percentage.

Compound decay (depreciation) is when the percentage is deducted after each time period.

Decay can be calculated using the formula, which is very similar to the formula for **growth**.

$$N = N_0 \times \left(1 - \frac{\text{percentage}}{100}\right)^n$$

N = amount after time period

N₀ = original amount

n = number of periods (this might be days/ minutes/ years)

Example: A new car is bought for £13,000. It depreciated by 21% each year for 10 years. How much value is lost after 10 years (to the nearest pence)?

1. Use the depreciation formula to find the value after 10 years.

$$N_0 = 13,000, n = 10, \text{percentage} = 21\%$$

$$N = 13,000 \times \left(1 - \frac{21}{100}\right)^{10}$$

$$N = 1230.8758 \dots$$

$$\text{Value after 10 years} = \text{£}1230.88$$

2. Calculate the value lost.

$$13000 - 1230.88 = \text{£}11,769.12$$

$$\text{Value lost} = \text{£}11,769.12$$

Compound interest

Compound interest is a form of **compound growth**.

Simple interest is where we increase an original value by a percentage.

Compound interest occurs when the **added interest then also receives interest** in subsequent periods.



Example: The compound interest rate at a bank 3% per year. Lauren invests £100 into the bank. How much is in her account after 3 years?

1. Work this out year by year.

$$\text{After 1}^{\text{st}} \text{ year: } 100 \times 1.03 = \text{£}103$$

$$\text{After 2}^{\text{nd}} \text{ year: } 103 \times 1.03 = \text{£}106.09$$

$$\text{After 3}^{\text{rd}} \text{ year: } 106.09 \times 1.03 = \text{£}109.27$$

Lauren has £109.27 in her account after 3 years

Interest is compounded annually; this is not the same as simple interest.

*If using **simple interest**, we would find 3% of £100 = £3.*

We would then add $3 \times \text{£}3$ to £100 = £109 after 3 years.

This is slightly less than the value found using compound interest, but for larger initial values the difference is more significant.

2. Alternatively, use the growth formula.

We can also calculate compound interest quickly using the growth formula.

$$N_0 = \text{£}100, n = 3, \text{percentage} = 3\%$$

$$N = 100 \times \left(1 + \frac{3}{100}\right)^3 = \text{£}109.27$$



Compound Growth and Decay - Practice Questions

1. Red Squirrels are entering the UK at a rate of 5.2% a year. Currently there are 590 red squirrels in the UK. What is the expected number red squirrels after 6 years?
2. UK retirees are migrating to holiday homes in Spain. Every year, 2.3% of UK residents move to Spain. In 2021, there are 2700 UK retirees. How many retirees will there be in 2027?
3. Water in a tank is leaking at a rate of 5.5% a second. The tank is filled up with 6 l of water. How much water is left after 8 seconds in millilitres?
4. A new car is bought for £15,000. It depreciates by 33% each year. Tim sells his car for the value after 3 years. How much did Tim lose?
5. Ethan loans £700 from a bank where the cost of borrowing is 5% per year. Calculate the extra amount of compound interest Ethan pays in 2 years.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.

